Paradox of Clarity: Defending the Missing Inference Theory

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The paradox. Barker and Taranto’s theory

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Question

*Why ever assert clarity?*
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Question

*Why ever assert clarity?*

If the evidence presented to every participant of the conversation (part of the common ground) already entails \( p \), there is no need in stating \( p \). The common ground, viewed as a set of possible worlds, does not change after the assertion of clarity is made.
Two theories examined by B&T:

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- Clearly, $p$ signals that the public evidence entails $p$. 
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- Deal with objections to the missing inference theory stated by B&T;
- Show how to formalize the missing inference story;
- Compare to epistemic must;
- Final remarks.
Problems with B&T

On B&T’s theory, assertion *Clearly, p* does not entail *p*. Instead, it guarantees that the speaker believes *p*. This explains why sentences like

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are anomalous. However once we change the tense, those pragmatic factors are no longer at work.

(2) #It was clear that Abby was a doctor, but in fact she was not.

is just as bad as the previous example, but B&T have no explanation for this.
(3) A and B are sitting in an emergency room. A woman \((D_1)\) in a lab coat walks along the corridor.

a. A: This is clearly a doctor.

A man \((D_2)\) walks by in the opposite direction. He wears a lab coat as well. He also has a stethoscope around his neck and carries a medical record under his arm.

b. A: Clearly, this is another doctor.
Contrary to Barker and Taranto’s claim, clarity assertions can be used in situations where there is no vagueness at all and the standards for belief/justification are completely determined. In particular, mathematical discourse:

(4) Take an integer $n$ divisible by 9. Clearly, $n$ is also divisible by 3.
(5) *It is clear to A from S that* $p$ *signals that A has performed a valid inference which has* $S$ *as premises and* $p$ *as conclusion.*
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This point of view is discussed and rejected by B&T under the label ‘missing entailment’ theory.
Reasons for their rejection:
▶ in some cases, inference is not enough to justify a clarity assertion
(6) John is a bachelor. ≠Clearly, then, John is unmarried.

(7) John ate a sandwich and drank a glass of beer. ≠Clearly, he ate a sandwich.
▶ often, there is no entailment.

(8) Abby is wearing a lab coat
    Clearly, Abby is a doctor.
    In fact, she might be a TV actress.
Barker's objections can be answered by specifying the type of inference that can trigger a clarity assertion:

- To account for (6) and (7), we need to claim that the inference should be nontrivial (perhaps a trivial inference is one sufficient for belief ascription);
- To account for (8), we need to allow defeasible inferences.
On the other hand, the missing inference theory deals easily with objections to B&T’s theory raised in the previous section:

- By asserting clarity, the speaker takes full responsibility for the validity of his inference — even if the inference is defeasible;
- In the case of (3), deducing the doctorhood of the second person is a separate inferencing act, even if it is easier than the first one;
- Math inference is no worse than any other kind.
Making the theory formal

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In order to represent inferences, we follow the idea from Duc (2001) and employ a version of dynamic logic, where an application of an inference rule by an agent constitutes an elementary action. The result of such an action is that the rule’s conclusion is added to the corresponding agent’s belief set.
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\langle Easy_a \rangle B_a \phi \land \neg \langle Triv_a \rangle B_a \phi
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$$\langle \text{Easy}_a \rangle B_a \phi \land \neg \langle \text{Triv}_a \rangle B_a \phi$$

One can use other criteria as well to characterize easy inferences, such as the number of steps.
For example, assume that conjunction simplification (CS) is a trivial rule, and universal exploitation (UE) and modus ponens (MP) are easy rules.
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Suppose an agent $a$ is in the following information state:

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In this state, it will be true that $B_a(N \text{ mod } 9 = 0)$
By applying rules UE and MP, a can achieve the state

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S_2 = \left\{ \begin{array}{l}
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the following formulas will be true in \( S_1 \):

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\langle UE_a; MP_a \rangle B_a(N \mod 3 = 0) \\
\langle (UE_a \cup MP_a)^* \rangle B_a(N \mod 3 = 0) \\
\langle Easy_a \rangle B_a(N \mod 3 = 0)
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Since \( \langle Triv_a \rangle B_a(N \mod 3 = 0) \) is false in this situation, (4) is true, according to our definition.
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Clearly vs. epistemic must

In vonFintel and Gillies (2007), an argument similar to mine is made with respect to the epistemic must, and a similar solution is proposed:

Epistemic modals signal that their prejacent is not directly settled by the salient kernel (where ‘kernel’ is a non-logically closed set of sentences – G. B).
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*Epistemic modals signal that their prejacent is not directly settled by the salient kernel (where ‘kernel’ is a non-logically closed set of sentences – G. B).*

However, *clearly* and *must* are not interchangeable.

- In the *clearly* construction, the existence of an appropriate inference is part of the assertion. Unlike *must*, *clearly* can take narrow scope with respect to operators like negation and tense.
  
  (9) It is not clear to me that Abby is a doctor, but she might be.
  
  (10) It was clear to me yesterday already that Abby is a doctor.
Must does not have to be based on public evidence, even when the relevant group is not specified explicitly.
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Certain types of inference can be marked by must, but not by clearly:

(11) John left two hours ago
    a. He must be home by now.
    b. ?Clearly, he is home by now.
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One can use clearly (but not it’s clear that) to signal an inference whose conclusion is already known to the speaker.

(12) Mary has been out of town for three days. She has not phoned. Clearly, I’m worried/ # I must be worried.
Final remarks

Barker and Taranto’s question ‘why ever assert clarity?’ receives a plausible explanation under our analysis: the speaker notifies the audience that the information they have is sufficient to infer $p$. Each member of the audience is invited to build the inference for themselves. The clarity statement can be used to build a greater confidence in the audience than simply stating $p$: upon deriving $p$, the hearer does not depend any longer on whether he trusts the speaker.
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(13) It is reasonably clear that Mars is barren of life.

While defeasible inferences can lead to varying levels of confidence in their conclusions, this is not represented in the formal system I am building.
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- Incorporating the evidence argument of clarity assertions:

  (14) It is clear from the way John speaks that he is disturbed. would require complicating the logic I am using.
Representation of sentences in an internal language, manipulated by inference, is a philosophically plausible idea (Fodor 1975 being perhaps the most famous exposition).
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References


